A LESSON THAT MAY ENHANCE CLASSROOM COMMUNICATION TO DEVELOP STUDENTS' MATHEMATICAL THINKING IN VIETNAM

DESCRIBING A REAL FACT BY A LINEAR EQUATION AND SOLVE IT

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In Vietnam, after launching the national standard mathematics curriculum in 2006, the classroom mathematics teachers have learnt more on the generative teaching strategies that encourage mathematical communication to implement more effective lessons focusing on mathematical thinking. Vietnamese mathematics teachers believe that classroom oral communication is of outmost importance for students learning mathematics. In particular, teachers' questioning, listening, and responding approaches in the classroom have been suggested to characterize their pedagogical practices, and to reflect their beliefs about mathematics and its teaching and learning. The aim of this paper is to examine a lesson that we considered may enhance classroom communication to develop mathematical thinking of primary students in Vietnam. A case study will be analysed using the observed students' activities in a videotaped lesson.

1. INTRODUCTION

In Vietnam, because the emphasis of old curriculum was on procedural knowledge and memorization of algorithms, students often worked independently completing exercises in the textbooks and workbooks. Students practiced mathematics independently to learn the mathematical concepts. When asking students questions, most teachers seek one "right" answer to the math problem and will explain why the answer is correct.

The reform curriculum supports active learning that creates opportunities for students' mathematical communication at different levels of thinking. Especially it supports students think about, discuss, extend, elaborate, verbalize, write, listen, read, and inquire about mathematical concepts. Communication is about participating, interpreting, and negotiating meanings, and it involves all classroom members alike.

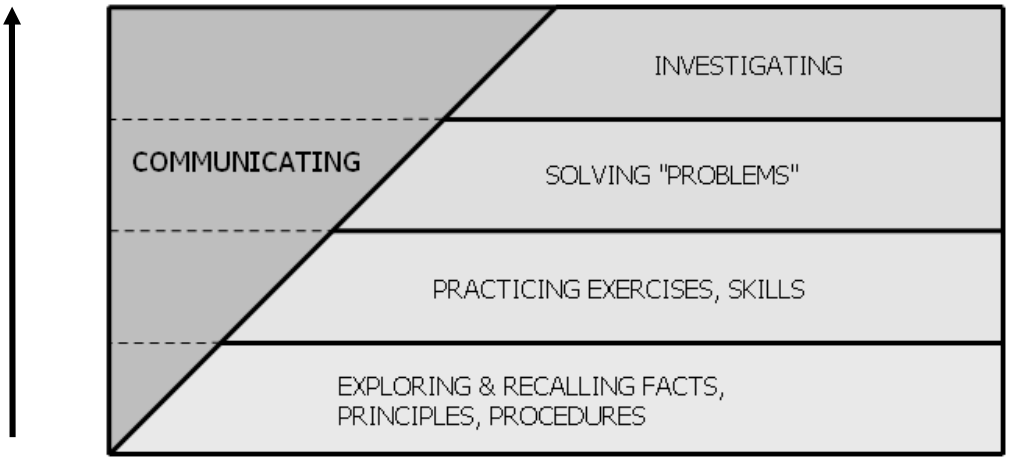


Figure 1. *Levels of thinking and opportunities for communicating at each level*

School reform mathematics education aims to help students achieve four following objectives: knowledge, skills, thinking and attitudes. Especially, with thinking objective, the mathematics curriculum provides opportunities for students to develop:

- The ability of observing, verifying, predicting, rational reasoning and logical reasoning;
- The ability of expressing precisely and clearly their own ideas and understanding the ideas of others;
- The characteristics of thinking, especially the flexible, independent and creative thinking.
- Thinking operations: comparison, analogy, generalization, and specialization.

The assessment of students' ability to communicate mathematics should provide evidence that they can express mathematical ideas by speaking, writing, demonstrating and depicting them visually.

2. TEACHING MODEL TO ENHANCE CLASSROOM COMMUNICATION IN MATHEMATICS

Normally in a mathematics class, students are not accustomed to talk about mathematical concepts. They are usually taught the concept by the teacher. Therefore, teachers need to use a number of approaches to probe students' thinking in mathematics. Students are not natural talkers in the mathematics classrooms. Older students will encounter more complex concepts in higher levels of math, discussing, talking, elaborating, writing, reading, and thinking about complex themes and concepts will help students to obtain deeper understanding in mathematics.

From the curriculum and textbooks we can categorize lessons into two main types as follows.

New lessons

a. Help students pose, explore and solve problems

Teacher facilitates students to explore and pose questions, problems of the new lesson when they engage in a problematic situation. Teacher then facilitates students to mobilize what they have experienced and learnt to recognize and seek for the relationships between the posed problem and their known knowledge to find the appropriate strategy to solve problem.

b. Give students opportunities to consolidate and apply knowledge constructed after learning new lesson such that students start to achieve the new knowledge

In the mathematics textbook at primary level, after new lesson usually there are three exercises for students to consolidate and practice what they have learnt in the lesson. The first two exercises require students know how to apply and practice directly the new knowledge. The third usually is a problem requires an indirect application of new knowledge.

Consolidation, practice and general practice lessons

a. Help students recognize learnt and new knowledge in various problems
b. Help students self practice on their pace of ability
c. Give students opportunities to help each other in small group with effective interactions by using worksheets.
d. Practice students having habits of checking, evaluating their works.
e. Practice students having habits of finding various strategies and choosing most appropriate strategy to solve problem.

Four main activities in a lesson

Four main activities in a lesson that teachers should follow to develop students' mathematical thinking (MoET 2006):

Activity 1. Examine and consolidate the previous knowledge involved with new lesson;

Activity 2. Teacher facilitates students explore mathematical knowledge and construct new knowledge by themselves.

Activity 3. Students practice the new knowledge by solving exercises and problems in the textbooks and exercise books.

Activity 4. Teacher concludes what students have learnt from new lesson and assigns the homework.

Each teaching and learning activity in classrooms gives opportunities for students' communication.

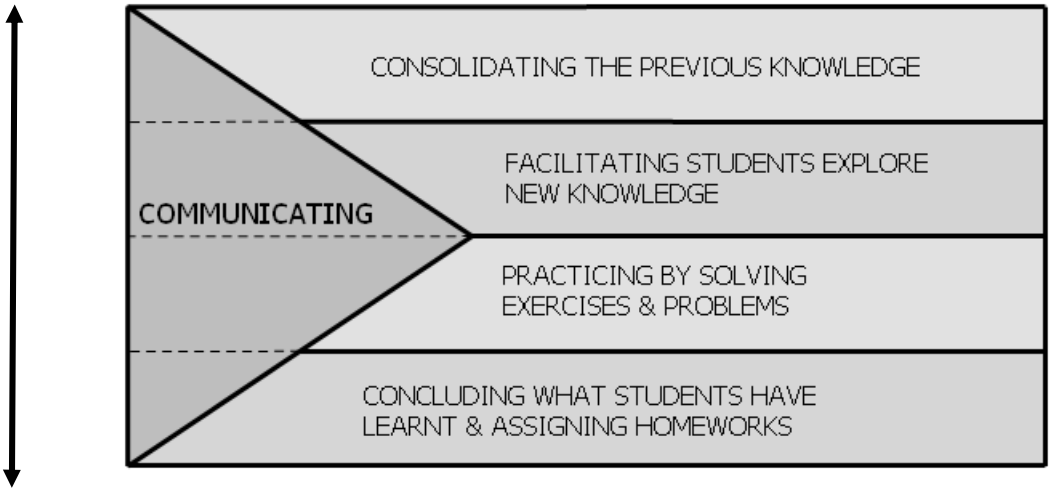


Figure 2. *Teaching activities and opportunities for communicating at each activity*

Engaging to the lesson, the students will have opportunities to show their mathematical thinking through:

- The ability of observing, predicting, rational reasoning and logical reasoning;
- Knowing how to express procedures, properties by language at specific levels of generalization by words, word formulas;
- Knowing how to investigate facts, situations, relationships in the process of learning and practicing mathematics;

Developing ability on analyzing, synthesis, generalization, specifying; and starting to think critically and creatively.

3. THE RESEARCH LESSON

The lesson which will be analyzed in this paper is prepared by classroom teacher for grade five primary students. We can find from the lesson plan the three main tasks and a consolidation task proposed in the lesson:

Introductory Task. A segment with the length of 90 cm is divided into 4 equal parts and remained 10 cm. What is the length of each part? Write another equation with the same x?

Task 1. You have a segment with the length of 90 cm drawn on your cartridge paper. You have a red rod representing 1 unit, one piece of blue rod representing unknown

variable x. By measuring the segment with your given unit, make an equation on x. Find x? From the discovered equation, write another linear equation with the same x?

Task 2. Write a linear equation with a "*minus sign*" at the second term.

Consolidation Task. Find x such that $5x + 2 = 17$. Then draw a diagram to illustrate the given equation.

At the end of Grade 4, students know how to solve some linear equations with variable x.

Example

Find x, such that:

$$34\,x = 714;\ 0.12\,x = 6;\ 4x + 10 = 90.$$

Usually students learn how to solve a given equation and never know that where the equations can come from. This study observes how students describe a real situation by an algebraic equation.

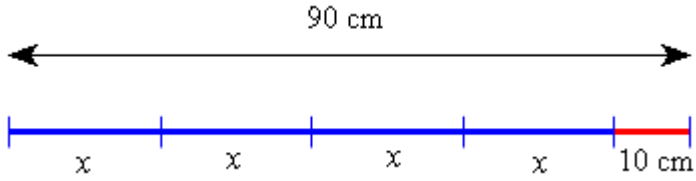
In the first semester of grade 5, if we set the problem in a different way:

A segment with the length of 90 cm is divided into 4 equal parts and remained 10 cm. Denote the length of each part be x. Find the length of the segment in term of x. What is the length of each part?

This problem is quite different with what students have learnt in grade 4. The problem requires them to write a linear equation for a real situation and then solve it.

4. ANALYSIS OF THE TASKS

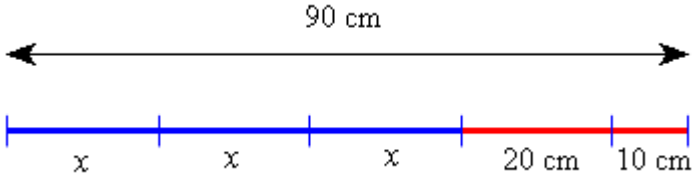
Analysis of Introductory Task. A segment with the length of 90 cm is divided into 4 equal parts and remained 10 cm. If we call the length of each part be x, find the length of the segment in term of x. What is the length of each part?



This task is an introductory activity. It requires pupils to make a linear equation and solve it as follows:

$$4x + 10 = 90 \square\ 4x = 90 - 10 = 80 \square\ x = 80 \square\ 4 = 20$$

From the equation $4x + 10 = 90$ obtained by pupils, the teacher ask them to write another equation with the same x and the sum still equals to 90?

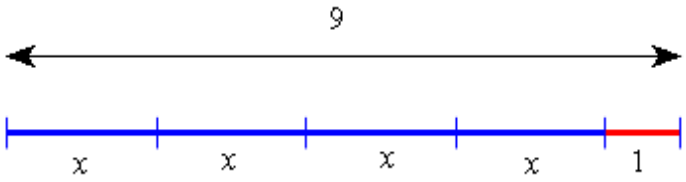


The pupils may reach to different answers such as:

$$3x + 30 = 90;\ 2x + 50 = 90;\ x + 70 = 90;\ ...$$

The aim of this introductory task is to help students create many linear equations by themselves for a real fact.

Analysis of Task 1. You have a segment of 90 cm. You have a red rod (10 cm) representing 1 unit, one blue rod representing variable x. By measuring the segment with your given unit, make an equation on x. Find x?



This is also an open-ended task that requires students to make a systematic list of all abilities. The total length is given but the number of variables *x* is unknown.

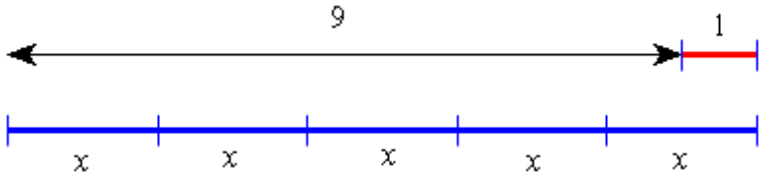
N. of unknown *x*	4	3	2	1
N. of units	1	3	5	7
Linear equations	$4x + 1 = 9$	$3x + 3 = 9$	$2x + 5 = 9$	$x + 7 = 9$

There are 4 or more answers to this task. Students know how to analyze a natural number into sum of two parts, first part is the number of variables *x* and second part is the number of units.

Analysis of Task 2. Write a linear equation with a "*minus sign*" at the second term.

In this task, teacher does not ask students to measure the segment by using given rods but encourages them to generalise what they have observed in some concrete situations above to create their own equations with a minus sign at the second term.

Students observe the situation: When does the total length of blue rods exceeds the segment one unit, and then write an equation for your measurement?



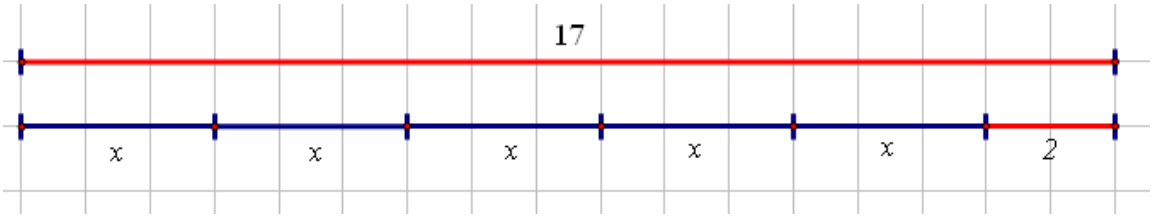
This is also an open-ended task that requires students to make a systematic list of all abilities. The total length is given but the number of variables *x* is unknown.

N. of unknown *x*	5	6	7	8
N. of units	–1	–3	–5	–7
Linear equations	$5x - 1 = 9$	$6x - 3 = 9$	$7x - 5 = 9$	$8x - 7 = 9$

There are many answers to this task. Students know how to analyze a natural number into two parts, first part is the number of variables *x* and second part is the number of units took away.

Analysis of Consolidation Task. Given a sheet of 1 cm ☐ 1 cm grid paper. 1 cm is one unit. Find *x* such that $5x + 2 = 17$. Draw a diagram to illustrate the above equation.

The aim of this task is to help students consolidate what they have learnt. They solve the equation $5x + 2 = 17$ and get $x = 3$ units. And then pupils draw the diagram as follows:



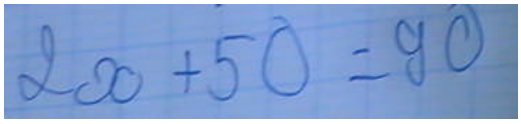
5. ANALYSIS OF THE VIDEOTAPED LESSON

The lesson is videotaped and analyzed using the video recording and the transcript. The actual lesson included several activities. The analysis in this section will be conducted by dividing the actual lesson into three stages: introductory activities, activities for task 1, activities for task 2, and activities for consolidation task. Each stage will be described and analyzed.

Introductory activities

1. Students were asked to find the length of each part if the segment is divided into three equal parts?
2. Students were asked to write an equation and find the length of each part if the segment is divided into four equal parts and remained 10 cm?
3. From the written equation $4x + 10 = 90$, students discussed in small groups of 4 students to list as many abilities as possible.
4. Students had to recognize the relationship between the number of unkown x and the number of units.

In the lesson, some students made only one equation as required and then stop working.



Teacher asked students to paste their answers on blackboard.

S: There are many answers to this question.

T: Can you check your answer?

S: My equation is $2x + 50 = 90$. We have $2x = 90 - 50 = 40$. Then $x = 40 \square 2 = 20$ (cm).

T: Can you write another equation with the same x and the sum still equals to 90?

S: $x + 70 = 90$.

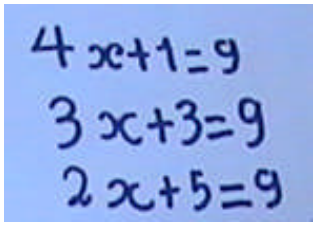
T: Can you find x?

S: $x = 90 - 70 = 20$.

Activities for Task 1

1. Students were asked to measure the segment of 90 cm by using 10 cm-red rods as units.
2. Students were asked to use some 20 cm-blue rods and 10 cm-blue rods to measure the length of the segment.
3. Students were asked to write the result of their measurements under an expression in terms of x?
4. Students discussed in small group of 4 students to write as many equations as possible.
5. Students had to recognize the relationship between the number of unknown x and the number of units.

In this task, first students used given 20 cm-blue rods and 10 cm-blue rods to cover all the segment. Then they reported their measurement by an equation, for example:



$$4x + 1 = 9.$$

Students solved this equation and got an answer $x = 2$.

From this equation, students decreased number of x to obtain another equations, such as:

$3x + 3 = 9$ or $2x + 5 = 9$.

This group missed the case $x + 7 = 9$.

T: What is the length of the segment if using 10 cm - red rod as unit?

S: The length of the segment is 9 units.

T: Using red rods and blue rods together to measure the length of the segment, how can you write the results of your measurement?

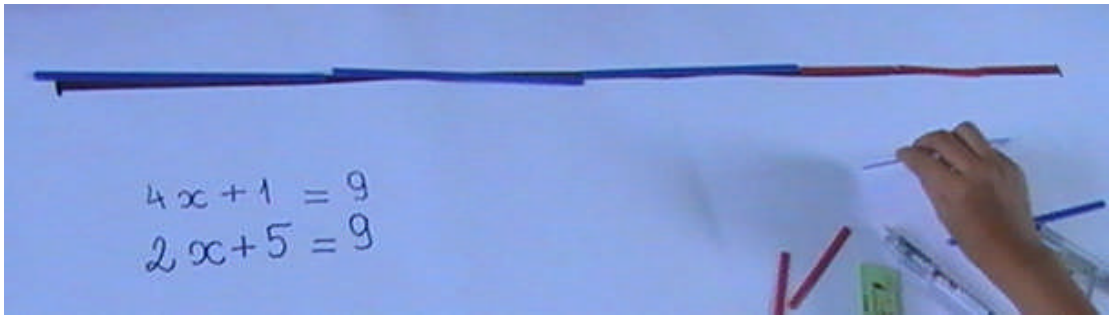
S: $4x + 1 = 9$. We could not use more than $4x$, it exceeds the given segment.

T: Can you find x?

S: $4x = 9 - 1 = 8$; $x = 2$.

T: From your equation $4x + 1 = 9$, can you write another equations by decreasing the number of x?

S: $3x + 3 = 9$.



T: Who can give another equations?

S: $x + 7 = 9$.

Activities for Task 2

1. Students were asked to write a linear equation with a "*minus sign*" at the second term.
2. Students used the 20 cm- blue rods to cover all the segment.
3. Students were required to create an equation for the first case that blue rods cover all the segment.
4. Students were asked to "*increase*" the number of blue rods (variable x) to create new equations.

S: There are 5 blue rods. And the equation is 5 *x* - 1 = 9.

T: From the result of your measurement for the segment, write another equations by increasing number of *x*.

S: When we increase one *x*, we have to take away 2 units. So we have 6 *x* - 3 = 9.

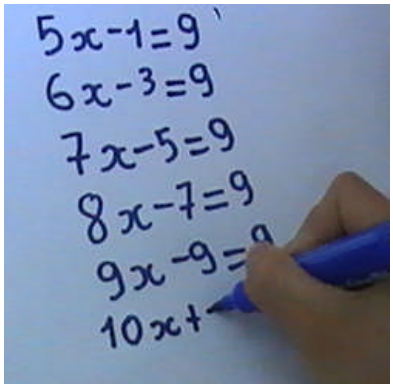
T: Can you check your answer?

S: We solve the equation and get *x* = 2.

T: Who can give another equations?

S: 7*x* - 5 = 9.

A group's answers:



Activities for Consolidation Task

1. Students were asked to find *x*.
2. Students were required to draw a diagram to illustrate the above equation.

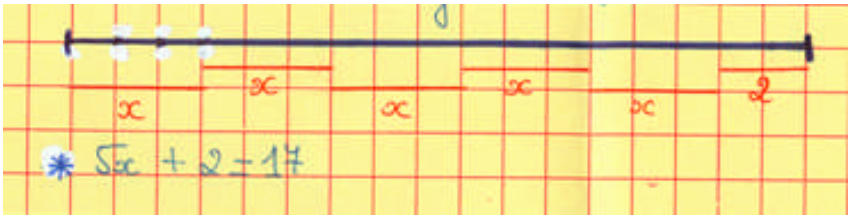
S: $5x + 2 = 17$

$5x = 17 - 2 = 15$

$x = 15 \square 5 = 3$

T: How can you explain this equation and your solution on the grid paper?

S: *x* = 3. And 5 ☐ 3 + 2 = 17. So we have the diagram:



6. DISCUSSION AND CONCLUSION

Communication is an essential part of the students' learning in mathematical classrooms, it helps students to achieve basic knowledge and develop mathematical thinking. With the reform curriculum, communications are required in the mathematics classrooms. Students may use verbal language to communicate their thoughts, extend thinking, and understand mathematical concepts. They may also use written language to explain, reason, and process their thinking of mathematical concepts. Communication is a tool which can help students to form questions or ideas about concepts. Most of teachers in Vietnam really need a practical framework to develop students' mathematical communication in their actual classrooms. Of course, each economy has slightly different approach in teaching mathematics to promote the mathematical communication, but we still have a lot of things in common to share and build up together a realistic framework that helps classroom teachers in each APEC economy.

This lesson was prepared by a senior teacher, he has involved in some educational projects at primary level. As I analyzed the activities in the lesson by using videotaped recording, the teacher followed four main activities in a lesson that were suggested by the MoET to develop mathematical thinking.

In introductory activities, the task is an open-ended task, it helped students get start to observe many abilities. Teacher managed students to work and achieved the following aims:

- Examine the students' previous knowledge in finding the answer for:
  $4 \times \square + 10 = 90$.
- Consolidate the previous knowledge involved with new lesson: Describe a problem related to the segment with the length of 90 cm by a linear equation.
- Explore new knowledge: create another linear equation from a discovered equation.

These activities gave students opportunities to show the ability of observing, predicting, rational reasoning and logical reasoning in creating new linear equations with the same solution with the original one.

In activities for task 1, teacher facilitated students explore mathematical knowledge and construct new knowledge by themselves. Students recognized the relationship between the fixed length of the segment and the numbers of variables *x*, units. They explored the linear equation $4x + 1 = 9$.

In activities for task 2, teacher facilitated students think algebraically. Students transferred from "*plus*" sign to "*minus*" sign to get another linear equation with negative number at second term. The students understood that the sign "*plus*: **+**" happens when the total of all *x* does not exceed to the length of the segment. The sign "*minus*: **–**" happens in another case.

In activities for consolidation task, students practiced the new knowledge by solving exercises and problems given by teacher.

These two tasks examined the thinking operations that occurred in the lesson such as: comparison, generalization, and specialization.

Teacher concluded what students have learnt from new lesson and assigned the homework.

Acknowledgement. This lesson study was conducted in Hue City, Vietnam under the collaborative framework involving mathematics education among the APEC Member Economies. Special thanks due to the principal, mathematics teachers of the primary school Le Qui Don, Hue City, Vietnam for their contribution to the research.

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MATHEMATICS LESSON PLAN

Grade 5 (10-11 years old)

Teacher: Senior Teacher Mr. Tran Quang Khen, Le Qui Don Primary School, Hue City, Vietnam.

1. **Title**: Describing a real fact by a linear equation and solve it.
2. **About the research theme**
   - Nurturing ability of observing, predicting, rational reasoning and logical reasoning in solving problems related to the describing a real fact by a linear equation and solve it.
   - Examining instruction that focuses on "understanding the relationship between variables and units and how to write a suitable linear equation for a real situation".
   - Examining the thinking operations that occur in the lesson such as: comparison, solving.

In this lesson we focused on equations of the form $ax = b$ and $ax + b = c$. The equation $ax = b$ is essentially a statement of division. Considering its construction and solution requires understanding how a student produces division, which entails understanding the student's multiplying schemes and multiplicative reasoning.

At the end of Grade 4, students know how to solve some equations with variable x.

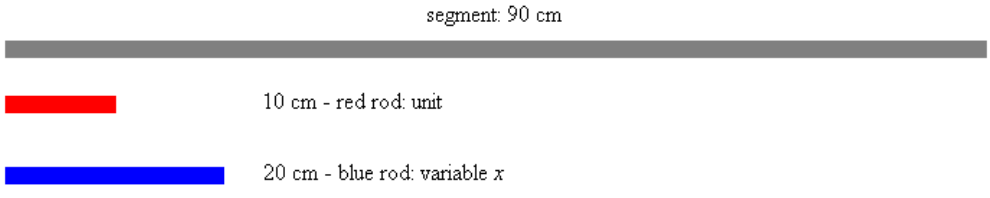
Example. Find x, such that: $40\ x = 25600$; $34\ x = 714$; $x - 7.2 = 3.9 + 2.5$.

3. **Goal**
   - For students to be able to write a linear equation for a real situation;
   - Know how to get another linear equation from a discovered equation.
   - Know how to solve and illustrate a linear equation on the grid paper.
4. **Instruction plan**
   - Understanding the relationship of two related quantities (variable and unit);
   - Identifying the left hand side of a linear equation.
   - Understanding how to "*decrease*" or "*increase*" the number of variables x to get new linear equation with the same solution.
5. **Instruction of the lesson**

   (1) Goal
   - For students to realize that making a linear equation for a real situation will help them understand the problem algebraically;
   - Look for a pattern or procedure to create a set of linear equation with the same solution.
   - Generalize the procedures obtained to create more linear equations.

   (2) Flow of the lesson

   Teacher prepares a segment of 90 cm, some red rods of 20 cm, and some blue rods of 40 cm.



Instructional Activities	Points for Consideration
Introductory Task. A segment with the length of 90 cm.	This task requires students to make a linear equation and solve it.
Q.0.1. The segment is divided into 3 equal parts. What is the length of each part?	The length of each part: 90 ☐ 3 = 30 (cm).
Q.0.2. If we call the length of each part be x. Write an equation to express the length of the segment?	$3x = 90$ $x = 90$ ☐ 3 = 30
Q.0.3. The segment is divided into 4 equal parts and remained 10 cm. What is the length of each part?	$4x + 10 = 90$ $4x = 90 - 10 = 80$ $x = 80$ ☐ 4 = 20
Q.0.4. From the equation $4x + 10 = 90$, can you write another equation with the same x and the sum still equals to 90?	$3x + 30 = 90$ $2x + 50 = 90$ ...
Task 1. You have a segment of 90 cm that drawn on your cartridge paper. You have a red rod (10 cm) representing 1 unit, one piece of blue rod representing variable x. By measuring the segment with your given unit, make an equation on x. Find x?	
Q.1.1. Using the red unit rod to measure the segment, how long is it?	The length is nine units.
Q.1.2. Using red rods and blue rods together to measure the length of the segment, how can you write the results of your measurement?	$4x + 1 = 9$ ...
Q.1.3. From the result of your measurement for the above segment, discuss in your group on how to write another equations by filling numbers in the blanks. $3x +$ ☐ $= 9$ $2x +$ ☐ $= 9$ $x +$ ☐ $= 9$ . . .	$3x + 3 = 9$, $2x + 5 = 9$, $x + 7 = 9$.
Task 2. Write a linear equation with a "*minus sign*" at the second term.	
Q.2.1. If the total length of your blue rods exceeds the segment one unit, write an equation for your measurement?	$5x - 1 = 9$

Q.2.2. From the result of your measurement for the above segment, discuss in your group on how to write another equations by filling numbers in the blanks. $6x - \square = 9$ $7x - \square = 9$ $8x - \square = 9$ ...	$6x - 3 = 9$ $7x - 5 = 9$
Consolidation Task Given a sheet of 1 cm ☐ 1 cm grid paper. 1 cm is one unit.	
Find x such that $5x + 2 = 17$.	$5x = 17 - 2$ $5x = 15$ $x = 3$
Draw a diagram to illustrate the above equation.